Bachelor's Thesis

Evaluation of Investment Strategies for Cryptocurrencies

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Declaration

I hereby declare, that I am the sole author and composer of my thesis and that no other sources or learning aids, other than those listed, have been used. Furthermore, I declare that I have acknowledged the work of others by providing detailed references of said work.

I hereby also declare, that my Thesis has not been prepared for another examination or assignment, either wholly or excerpts thereof.

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Signature

Abstract

How to invest money most profitably is not only a field of research, but also a question that most people are confronted with at some point. The easiest approach is to invest the money and leave it there for an arbitrary time period. This is a common approach, not only for a savings account, but also for assets like crypto currency or stocks. Contrary to investing there is also day trading, where the asset is bought and sold within daily or sub-daily time frames. The aim of this approach is to generate a higher profit than simply investing would yield. Technical Analysis is a field which provides a lot of day trading strategies that are easy to interpret and easy to implement. The question we ask in this thesis is, if we can use one of the most popular day trading strategies to get a higher return on the used capital than if we would invest traditionally. This strategy is called Simple Moving Average Crossover (SMAC). To answer this question we will study the related literature on this topic, construct an asset-pricing model under which the SMAC strategy would be optimal and then perform a hypothesis test on three different asset classes. Literature and the constructed model do not provide any reasoning why the strategy would incur a higher return than a traditional investment. This causes us to hypothesise, that the strategy does not yield a higher return. By performing a one-tailed binomial test for every possible parameter setting of the SMAC strategy, we find that this hypothesis is rejected for at most half of all possible settings. Thus, if we choose one setting randomly, we have a chance of less than 30% that the SMAC strategy is significantly better than an investment. But there are also settings which have been recommended

by self-proclaimed experts. Our tests show that if we chose one of these settings at random, in most cases we would have an even worse chance of using a SMAC setting that outperforms an investment.

Zusammenfassung

Die Frage, wie man am profitabelsten Geld investiert, ist nicht nur eine Frage in der wissenschaftlichen Forschung, sondern auch eine Frage mit der die meisten Menschen im Laufe ihre Lebens konfrontiert werden. Die einfachste Herangehensweise ist das Geld zu investieren und es dort für einen beliebigen Zeitraum zu lassen. Das ist ein üblicher Ansatz, nicht nur für ein Sparkonto, sondern auch für Kryptowährungen oder Aktien. Im Gegensatz dazu, gibt es das sogenannte "Day Trading", bei dem täglich oder in kürzeren Zeiträumen das Asset ge- und verkauft wird. Das Ziel dieses Vorgehens ist es, einen höheren Profit zu erzielen als bei einer einfachen Investition. Technische Analyse ist ein Bereich der viele Day Trading Strategien hervorgebracht hat, welche sowohl einfach zu interpretieren als auch einfach zu implementieren sind. Die Frage die wir in dieser Thesis stellen ist, ob wir eine der populärsten Day Trading Strategien verwenden können um eine bessere Rendite zu bekommen als bei einer herkömmlichen Investition. Diese Strategie ist die "Simple Moving Average Crossover" (SMAC) Strategie. Um diese Frage zu beantworten werden wir die Literatur zu diesem Thema untersuchen, ein Preismodell konstruieren bei dem die SMAC Strategie optimal wäre und dann einen Hypothesentest für drei verschiedene Assetklassen durchführen. Die Literatur und das konstruierte Modell geben keine Begründung warum die SMAC Strategie einen höheren Ertrag bieten sollte als eine herkömmliche Investition. Das veranlasst uns dazu die Hypothese aufzustellen, dass die Strategie tatsächlich keine höhere Rendite einbringt als eine Investition. Durch einen einseitigen Binomialtest für jede mögliche Parameter Einstellung der SMAC Strategie wird ersichtlich, dass

die Hypothese für maximal die Hälfte aller möglichen Einstellungen abgelehnt wird. Das bedeutet, wenn wir die Parameter zufällig wählen hätten wir eine Chance von weniger als 30% eine Einstellung zu wählen bei der die SMAC Strategie signifikant besser wäre als eine Investition. Aber es gibt auch Parameter, die von selbsternannten Experten vorgeschlagen werden. Unsere Tests zeigen, dass falls wir zufällig eine dieser Einstellungen wählen, wir in den meisten fällen eine noch schlechtere Chance hätten eine SMAC Einstellung zu verwenden die besser als eine Investition ist.

Contents

List of Figures

List of Tables

List of Algorithms

1 Introduction

How to make a profitable investment in a market, where prices seem to move random is not only a field of research, but also a question that many people ask themselves. For a long time, savings accounts were a common approach. They offer a constant interest with virtually no risk. But in recent years, interest rates have been falling and banks as well as their customers try to find viable alternatives. One such alternative, even though it includes more risk, is investing the money in a market. But this is by far the only reason why people would be interested in buying stocks, gold or crypto currencies.

The pension funds in Germany have been in serious dilemma. Because of the demographic structure, there are more people retiring than there are young people entering the work force. This means, that more people start to require money from the pension funds, while there are fewer people who work and pay into the funds. Poverty among seniors is a much debated topic in politics and there are even conservative German politicians telling seniors to invest in the stock market. Recent ventures to counteract this problem, like the "Riester-rente" were arguably unsuccessful. German financial institutions are well aware of this fact and try to offer new alternatives, like the "UniRBA Welt 38/200" at which we take a closer look in this thesis. It is a funds based financial product, where the invested capital is used to buy and sell the stocks in this fund according to the Simple Moving Average Crossover strategy.

Aside from stock, there is another asset class which has become more and more popular in recent years: Bitcoin.

Bitcoin is a crypto currency, which essentially means that a blockchain is used to keep track of the transactions. The most important difference to other asset classes, is that there is no institution which backs the value of the Bitcoin. Stocks for example, are backed by their respective company and currencies are backed by the government that issues it. Thus, Bitcoin has no inherent value. If there are no people willing to offer money for a Bitcoin, then it has no value. In that way, it is much more similar to gold or oil than stock or traditional (fiat) currency.

The price for one Bitcoin has increased from about 200 US Dollars in late 2017, to about $20,000$ USD one year later. In financial markets it is uncommon to have an asset which increases its value by a factor of 100 in a matter of months. Usually such a price behaviour is indicative of a price bubble. One popular example is the Dot-com bubble, which burst in the late 1990's. However, the Bitcoin bubble did not burst and after the price per Bitcoin hit the all-time high of $20,000$ USD, it managed to stay above $3,000$ USD. So, Bitcoin could provide to be a profitable investment opportunity. Assume we were determined to make an investment into Bitcoin, should we simply invest our money and wait or should we an investment according to a trading strategy like the one described above? In order to make that decision, we need to closely examine the strategy, which originates from the so called "day trading".

A day trader is a person or bot, who regularly examines the price history and decides whether to buy some asset, sell it or to do nothing. Usually the periods between examinations is fixed and can range from one day, down to one hour. Some traders rely on Technical Analysis, which may sound like a scientific field, but it should be emphasised that this is not the case. In Chapter 2, we will see how self-proclaimed Technical Analysts reason for the profitability of their strategies, as well as scientific research on this subject. It will become apparent, that scientific evidence in favor of the profitability of Technical Analysis is sparse.

In this thesis, we will pay special attention to the Simple Moving Average Crossover (SMAC) strategy. It is vastly popular among day traders, and as described above, even established financial institution sell products based on this strategy. This strategy requires two parameters. As we will later see, the choice of these parameters has a direct impact on the strategy's profitability. Proponents of this strategy recommend parameters, which we will take into consideration.

Another important strategy which we look at, is the Buy and Hold (BnH). It models the usual behaviour of someone making an investment, meaning all it does it buying an asset right at the start and nothing else. We will use this strategy to measure the performance of the SMAC strategy by comparing their return on investment. If the SMAC strategy yields a higher return on investment than the BnH, it would be reasonable to day trade using the SMAC strategy, instead of making a regular investment. Because the SMAC strategy is easy to calculate and interpret, we can facilitate day trading by simply deploying a trading bot.

But before we look at an empirical evaluation, we will examine in Chapter 3 an asset-pricing model for which the SMAC strategy is more profitable than the BnH. By doing this, we hope to find a mathematical justification or reasoning, why one should use the SMAC strategy. If we do not manage to gain this insight, it might be because using the SMAC strategy simply has no merit.

To prove this empirically, we will hypothesise in Chapter 4 that the SMAC has a higher return of investment than the Buy and hold with a probability of at most one half. This would mean, that the SMAC strategy is not consistently more profitable. We will test this hypothesis, by randomly sampling prices and performing a binomial test. The tests will be performed on crypto currency price data, like Bitcoin and Ethereum, but also stock prices and foreign currency exchange rates. One problem we face with this approach, is how we should choose the SMAC strategy's parameters. A common approach is to use some kind of hill-search algorithm to find the best parameters. Instead, we will exhaustively test all parameter choices up to a limit that is way above of what a day trader would usually use. By doing so, we see for which parameters we can reject the hypothesis. We will examine, whether the proposed settings produce a more profitable strategy or not. If they do not, the only option we have is to choose the parameters randomly.

2 Related Work

Trading strategies based on Technical Analysis has been a topic of debate over the years. They are favored because they usually do not require a mathematical or economical understanding and as such are easy to understand and implement. However they are criticised for their lack of scientific justification. We we will see how those in favor try to justify Technical Analysis, as well as the most cited arguments against it: the efficient market Hypothesis and the random walk hypothesis.

2.1 The Efficient-Market Hypothesis

In 1970, Malkiel and Fama published a paper about the review of the efficient-market hypothesis [1]. This work is widely regarded as consequential in the debate on market predictability. In 2013 Fama was awarded the Nobel prize in economics for his empirical study of financial markets. A market is called efficient if, at every point in time, it reflects all available information at that point. The Efficient market hypothesis simply states, that capital markets are efficient. This is a very vague definition, since it does not specify what constitutes information in this context and under which circumstances it can be considered available. In order to specify, an asset-pricing model is needed. Fama examines several models that have been conjectured to accurately describe the market. By reviewing empirical literature, the author finds extensive evidence in support of the hypothesis.

In a later work [2], Fama stated the joint hypothesis problem in regard to the efficient-market hypothesis. The problem arises from the fact, that the efficient market hypothesis does not specify a model for the market. So in order to test the hypothesis, a model is needed which closely describes the market. However, if we were to find empirical evidence which refutes the hypothesis we would not be able to tell if that is due to a false hypothesis or an erroneous model. This implies that is not possible to falsify the efficient market hypothesis, which makes it arguably unscientific.

2.2 The Random Walk Hypothesis

Another important hypothesis in regard to TA is the random walk hypothesis. Fama wrote his dissertation about this topic, providing one of the first extensive examinations of this hypothesis [3]. It states that the price of an asset follows a random walk. This is the case when the change in price is given by a stochastic process, where the random variables of the process are independent of each other and identically distributed.

Especially the independence of changes in price is problematic for TA, because it implies that it is not possible to draw conclusions about future prices from past prices. In his work, Fama finds for stocks of the Dow Jones Industrial Average that there is little to no dependence between changes prices. As an implication of this, he proposes portfolio analysis as a more rational approach to investing. We will examine this approach closer in 2.3.1. Also independence of price changes is not the end of technical analysis, as its proponents allege. We will examine their claim closer in 2.4.

2.3 Theoretical and Empirical Analysis of Trading Strategies

In this section we will see how trading strategies are evaluated in literature, as well as theoretical and empirical analysis of strategies based on technical analysis.

2.3.1 Modern Portfolio Theory and Theoretical Analysis

Modern portfolio theory (MPT) provides a framework founded in financial mathematics for the theoretical analysis of trading strategies. It makes assumptions about the market in order to provide a model. This model can then be used to compute the expected returns of some strategy. It is usually assumed, that there is one risk-less asset which has the purpose to secure funds (e.g. a stable currency) and one or more risky assets, which can be traded for profit. Contrary to technical analysis, MPT aims to build a portfolio, i.e. a collection of assets, such that the risk of a loss is minimized and expected returns are maximized. However, TA can be seen as a special case of MPT, where there is one risk-less asset and only one risky asset. This suggests that it is possible to examine TA by the means of MPT.

A good example for the general MPT approach is the work by Dhaene [4]. Here it is assumed, that the market is modeled by a Black and Scholes setting. This setting makes several assumptions itself, for example that the change of price of risky assets follow a Brownian motion stochastic process. That assumption conforms with the random walk hypothesis, which is usually accepted in the context of MPT.

We can see this approach toward analysing TA in a paper by Lorig, Zhou and Zou [5]. The authors analyse the exponential moving average crossover strategy from TA, which is similar to the strategy we examine in 3.4. They assume that there is one risk-less and one risky asset and (like Dhaene) that the price follows a geometric Brownian motion. For different kinds of drift they are able to show that the strategy is optimal.

However, this approach is problematic, as it provides not much insight as to why the strategy is profitable at all and when it should be used. Our approach will differ, in that we will construct a model where we are able to tell why the examined strategy is optimal and under which circumstances.

2.3.2 Empirical Analysis

There have been lots of studies on the profitability of TA-based strategies, but of varying scientific value. Especially data snooping bias is a problem in a lot of these works where, in short, a strategy is deemed profitable based on a price sample and is then tested on the same sample, obviously proving the strategies' profitability.
Park and Irwin conducted a review of the empirical literature on TA, where they acknowledge the problem of manipulation and put the testing procedures under scrutiny [6]. They have surveyed a total of 137 studies on the subject, from 1960 to 2004 and note that there has been a surge in the amount of papers on this topic in the last decades. About half of all examined works stem from the years 1995 to 2004. The authors distinguish between early and modern studies, because of their differences in statistical tests, examined strategies and data snooping bias. They find in early studies from 1960 to 1987 limited evidence for the profitability of TA strategies. But it should be noted, that these studies have much less robust testing procedures than modern studies. For example, parameters of trading strategies are not optimized and only a small number of different strategies is used. Out of 95 modern studies 56 find evidence in support of TA strategies, 20 find evidence against these strategies and 19 yield mixed results. Although this might suggest that TA might indeed be profitable, the authors note that the amount of positive returns may differ throughout different markets and times. Especially in the foreign exchange market TA seems profitable.
For an empirical evaluation, it is important how the strategy's parameters are chosen. In the studies evaluated by Park and Irwin, the parameters are optimized, in order

to find the most profitable parameters for a strategy. Our approach will differ: we will not optimize the parameters, but instead try a "brute force" approach. For this, we evaluate all possible parameter settings, up to the highest recommendation by the advocates for TA (see Section 2.4).
A common metric to compare trading strategies is the return of investment (ROI). In most studies, it is assumed that there is an underlying model for the price data. With this model, a statistical test is performed to determine, whether the ROI of some strategy is significant or not. As explained, we will not assume that asset prices in general are distributed according to some model. Instead of testing whether the ROI is significant, we use most likely the simplest investing strategy (Section 3.3) and compare its ROI against that of the strategy we want to test. We will do this for a number of random samples to determine whether the strategy is better than the baseline in a significant number of trials.

2.4 The Technical Analysts' Arguments

In this section we will examine the arguments of self-proclaimed Technical Analysts in favour of their strategies. However it should be noted that technical analysis is not a scientific field: there are usually no agreed upon definitions or interpretations of strategies, much less consistent justifications. One of the most popular advocates is Pring, who wrote a book where he aims to explain technical analysis [7]. Specifically his explanations for two strategies examined in this thesis are relevant.
For the Simple Moving Average Crossover (SMAC) strategy Section 3.4 the author states how to calculate an SMA and how to interpret the crossover of two SMAs. He also gives "recommendations" for parameters, but aside from that there is little explanation for this strategy. He simply states, that a SMA with small sample size approximates a short-term trend, while a bigger sample size SMA does the same with a long-term trend. However there is no proof, no asset-pricing model under which this statement would be true and not even empirical evidence. The only justification

the author has to offer, is a price chart showing how this strategy is supposed to work, which is hardly a scientific approach. It also should be noted that he does not recommend some choice of parameters. He states that there is no perfect setting and that a good choice depends on trial and error.

Another source to consider on this subject is a subsidiary insurance company “R+V Versicherung” of the well established German bank Volksbank. One of their products is the “UniRBA Welt 38/200” [8]. They use the SMAC strategy as a way to reason for the safety of their investment product. However they too, do not provide a justification, neither for their strategy choice nor for their choice of parameters.
A source that provides a little more concrete information on parameter choice is the website “Investopedia.com” [9], which is often referenced within the crypto-trading community. Even though the name bears similarity to Wikipedia, it should be noted that Investopedia is not edited by a community, but by a fixed board of editors. One article on the site refers to SMAs and common settings [10]. These are the sample sizes 10, 20, 50, 100 and 200. These sample sizes define 10 possible settings for the SMA Crossover strategy. They note that especially the SMAC strategy with sample-sizes of 50 and 200 is promising. This is because a cross between a 50-day SMA and a 200-day SMA is referred to as Golden or Death Cross [11], depending on the direction in which they cross. If the SMA with the shorter sample-size crosses above the other SMA from below, it is considered to be a indication that the price will rise and the crossover is called a Golden Cross. Conversely, if the SMA with shorter sample-size crosses below the other SMA from above, it is interpreted as an indication that the price will fall and the crossover is referred to as a Death Cross. In Chapter 4 we will examine these settings closer. Aside from this information the website has little to offer, aside from an explanation on how to apply this strategy.

3 Theoretical Analysis

In this chapter we will see what constitutes a trading strategy and under which circumstances each of them is expected to be profitable and/or optimal.

3.1 Price Data

Before we can examine the strategies, we will need to see how price data is structured. This data is not only used as input for some strategy, but also to evaluate the performance of the strategy. The difficulty with price data is that it can change in infinitesimally small time steps. Every transaction moves the price a little bit up or down. Thus, if we consider the price data simply as prices at discrete points in time, we would loose the information of price changes between the discrete points in time. For example, if we look at daily prices we would not be able to tell if the price rose and fell again within a day.

This problem is partially solved by using OHLC-data (also called "candles"). OHLC stands for Open, High, Low and Close. This type of data assigns these four values to discrete points in time. Note, that each point in time also defines an interval that starts at this point and ends right before the next point. The values Open and Close specify the respective opening and closing price for this interval and High and Low specify the maximum and minimum price during the interval. We will call the size of these intervals the granularity of the data.

For example, let us assume we examine price data with the granularity of one day.

Then Open and Close value would give us the first and the last price of each day, while High and Low would give us the highest and lowest price of each day. This raises the question, whether the closing price for one day is also the opening price for the next. This is not necessarily true for all assets. Some stock exchanges do not operate during Saturdays and Sundays, but the stock price may still change during that period. Thus the closing price for Fridays can differ from the opening price on Sundays. In the case of cryptocurrencies, such differences between opening and closing price should not occur, as most exchanges operate at all times. However cryptocurrency exchanges may still produce data with these differences, either because prices change very rapidly by a great amount or simply because of an error in the recording.

TA assumes this kind of data as input for its strategies. As a consequence, they are designed to act once at every time step of the input data. For example, a strategy that is applied to daily OHLC-data will give a recommendation to buy, sell or do nothing at the end of every day. It is important that the strategy gives its recommendation at the end of the period, because the High, Low and Close values are not definite until then. This recommendation will be acted upon as soon as possible.

The strategies we will discuss in this thesis only require the closing price, the other three values of the OHLC-data is not relevant. We assume that these prices are given by some sequence p, ranging from index 0 to t:

$$p = p_0, p_1, \ldots, p_{t-1}, p_t \tag{1}$$

3.2 Trading Strategies

A trading strategy is an algorithm that generates Buy or Sell signals (recommendations) based on several inputs. The strategies in this thesis require the price history up to a certain point, as well as differing configuration parameters. The generated signals could theoretically specify an amount of asset that should be sold or bought. But this is only reasonable when a strategy has some kind of risk-adjustment mechanic, which the strategies examined here do not. So when one of them sends a Buy or Sell signal all of the available assets will be transferred. However, there are different means to employ risk-adjustment for arbitrary strategies, as we will see later.

In order to evaluate trading strategies (theoretically as well as empirically) we need to clearly define what happens if a Buy/Sell signal is sent. First, we will assume that there is one risky and one risk-less asset. The risky asset is the cryptocurrency we want to trade and the risk-less asset is some stable currency that we can exchange against the cryptocurrency. This could be a "fiat" currency (fiat meaning it is established), e.g. US Dollar, Euro, etc., or a so called "Stablecoin", which is a cryptocurrency that is supposed to stay at a constant value.

If, for example, a Buy signal is sent by the strategy we can interpret this as a recommendation to buy the underlying risky asset (the cryptocurrency). Aside from the price, there may also be a transaction fee. Different cryptocurrency exchanges have different means of applying these fees. One common approach has been adapted from foreign exchanges: the spread.

The spread is the difference between buying and selling price of an asset. By setting the buying price higher than the selling price, the exchange can make profit. Essentially, if (different) traders were to buy and sell asset at the same time, the exchange would make profit proportional to the spread. In reality, this mechanic is more complex and depends on other factors, like trading volume of the asset or the rate of change of the price. The exchange will adjust the spread according to these factors, such that the spread may change over time. However, this change is small and usually the spread stays between one half to one percent. That means that the buying price is higher than the original price by that percentage and equivalently the selling price is lower than the original price by that percentage. For example at Coinbase.com, they state that their spread is about one-half to one percent [12]. The Bison app, one of the few exchanges based in Germany and available with a German

bank account, uses a spread of about 0.75 percent [13]. Aside from the spread, some exchanges (like Coinbase) charge other fees. These fees may depend on the size of the transaction, payment method and other factors. For simplicity we will assume that the only fee we have to pay is the spread.
A strategy can either send a Buy, Sell or No-Operation signal. Later, we will numerical values to describe these signals. A Buy signal is represented by 1, a Sell signal by -1 and a No-Operation by 0.

Measuring the performance of a strategy is done by calculating the return on investment (ROI). The ROI is the ratio between the acquired profit (or loss) and the initial investment. We will assume that the only possibility to get returns is by exchanging the asset. There are other means to get returns. For stocks there may be dividends and in the case of cryptocurrencies, these means could be mining or initial coin offerings. For simplicity sake and to be able to compare the strategies for different asset classes, we will neglect additional returns.
We can try to increase the amount of available funds by buying the cryptocurrency and then selling it at a later point in time. This is called a trade. A trade can either have a positive or a negative return and has an entry and an exit. The entry is the point in time at which the asset was bought and conversely the exit is the time when it was sold.
The ROI of a single trade can be calculated from the available funds before and after the trade. Let b be the entry time and s the exit time. The funds at some time t are denoted as F_t and the price of the asset as p_t. The spread is assumed to be constant and is denoted by c. At time b we exchange our funds F_b against some amount of asset $\frac{F_b}{p_b \cdot (1+c)}$. This is the entry of the trade. When we exit at time s, the bought assets are sold again for price p_s (with cost c). Thus the amount of funds after the trade is given by:

$$F_s = \frac{F_b}{p_b \cdot (1+c)} \cdot p_s \cdot (1-c) = F_b \cdot \frac{p_s \cdot (1-c)}{p_b \cdot (1+c)} \tag{2}$$

From F_b and F_s we can calculate the ROI R of the trade:

$$R = \frac{F_s - F_b}{F_b} = \frac{F_b(\frac{p_s \cdot (1-c)}{p_b \cdot (1+c)} - 1)}{F_b} = \frac{p_s \cdot (1-c)}{p_b \cdot (1+c)} - 1 \tag{3}$$

So in order to compute the ROI of a trade, we only need the price at point b and s. The strategies will generate a series of one or more trades by sending Buy and Sell signals. In the case of the strategies presented here, each one can not have multiple concurrent trades. Let T be the set containing the entry and exit times of the trades, such that $T = \{(b_1, s_1), (b_2, s_2), \cdots\}$. Then we can calculate the ROI R of some strategy by calculating the product over the ROIs of the individual trades:

$$R = \left(\prod_{(b,s) \in T} \frac{p_s \cdot (1-c)}{p_b \cdot (1+c)} \right) - 1 \tag{4}$$

As we can see, knowing the actual amount of funds is not necessary to calculate the ROI of a strategy. Only the price history, transaction cost and entry/exit times of the trades are needed. Also the strategies that we will discuss do not take the amount of available funds into consideration when generating signals. Thus, in order to calculate the ROI of a strategy, we do not need to specify the amount of funds a strategy receives or has available.

We also need to define, when a strategy is optimal for some sequence of prices. Assume some strategy generates a set of trades T for a price sequence $p = p_0, \ldots, p_t$. We call this strategy optimal with respect to p, if there is no other set of trades T' which has a higher return. This means, that the following condition must hold for all T':

$$\left(\prod_{(b,s) \in T} \frac{p_s \cdot (1-c)}{p_b \cdot (1+c)} \right) - 1 \geq \left(\prod_{(b',s') \in T'} \frac{p_{s'} \cdot (1-c)}{p_{b'} \cdot (1+c)} \right) - 1 \tag{5}$$

This definition can be problematic. Suppose the price is increasing from point 0 to point t and falling from point t to $t+1$ (see Figure 1). By our definition, a strategy would only be considered optimal, if it sold at point t before the price falls. However, the prices up to price p_t do not indicate that the next price p_{t+1} will be smaller than its predecessor. The earliest signal we can expect from a strategy that is based on price history alone, is at time $t+1$. Because all of the strategies we examine in this thesis are only based on price history, we will expand the definition in Equation (5). If there is exactly one set T', where each trade (b', s') has a corresponding trade (b, s) in T, for which the equations $b' = b - 1$ and $s' = s - 1$ hold, then we still call T optimal.

It should be noted, that there is no standard definition for trading strategies based on Technical Analysis. We will try to use the simplest, most intuitive and as such the probably most popular definition.

3.3 Buy and Hold

The Buy and Hold strategy is one of simplest strategies possible. It models the behaviour of a long-term investor, who buys some asset and then basically waits. One example where this strategy is commonly employed would be a savings account. Especially in the cryptocurrency-trading community this strategy is important. There, it even has gained its own name: “Hodl”, which stems from a typo in a forum post. It is based on the sentiment that the prices of Bitcoin (and potentially other cryptocurrencies) will rise exponentially eventually.
In this section, we will first see an exact definition for this strategy and then the optimal model.

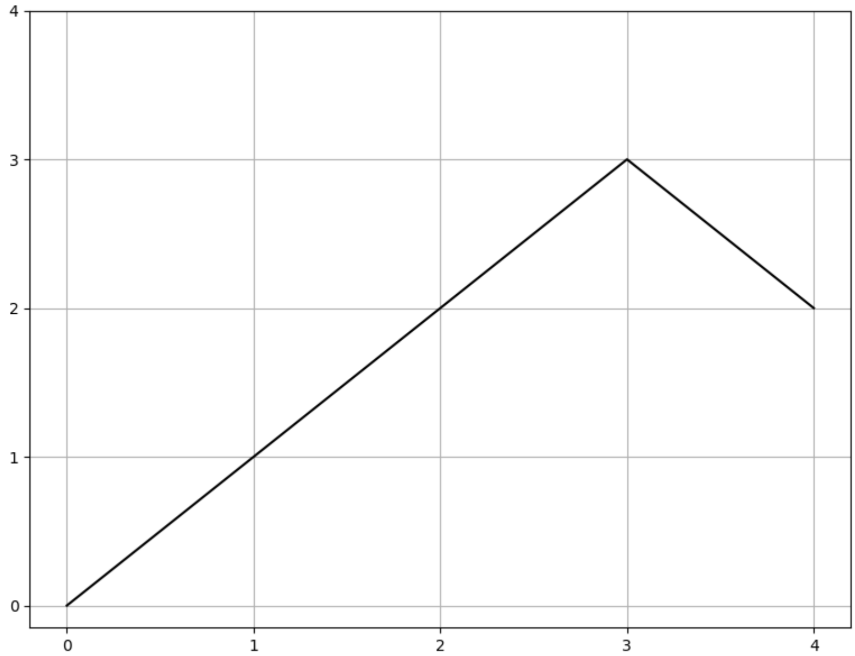

Figure 1: Example price history, where strategy optimality is not obvious In this example: $t = 3$. If prices were only known up to fourth one, sending a Sell signal based on price history would be irrational

3.3.1 The Strategy

The Buy and Hold strategy will always produce a Buy signal as its first and only action. Hence the name, after buying it will “hold”, i.e. solely send No-Operation signals.

Algorithm 1 Buy and Hold Strategy

procedure BuyAndHold($p = [p_0, p_1, \ldots, p_n]$)
    signal = [1, 0, 0, . . .];
    return signal
end procedure

The run time of this algorithm is constant.
In order to evaluate this strategy, we will assume that the assets being held are sold for the last price available at time t. Because there is only one trade ($T = \{(0, t)\}$), we can compute the ROI as follows:

$$R = \frac{p_t \cdot (1 - c)}{p_0 \cdot (1 + c)} - 1 \tag{6}$$

Intuitively the return is positive, if the last price p_t is higher than the first price p_0, else it is negative.

3.3.2 The Model

This strategy is optimal, as long as the following holds for $p_0 > 0$ and for all $i > 0$:

$$p_i \geq p_{i-1} \tag{7}$$

We can see that this model is optimal for the Buy and Hold strategy by assuming that there is some other strategy which may send its signals at any point other than the Buy and Hold.
Any strategy that does not send a Buy signal at time 0 will have to send its Buy signal at some time $b > 0$. Equivalently, this strategy will have to send its Sell signal at some time $s < t$. Assuming this strategy only produces one trade, it will have the ROI:

$$R = \frac{p_s(1 - c)}{p_b(1 + c)} - 1 \tag{8}$$

If we assume that the price is given by the model above, we get:

$$p_0 \leq p_b \leq p_s \leq p_t \tag{9}$$

We can see that in this case, the buying price of the other strategy is higher than that of the Buy and Hold strategy. Conversely the selling price of the other strategy is lower than the selling price of the Buy and Hold. So we get:

$$\frac{p_t(1-c)}{p_0(1+c)} - 1 \geq \frac{p_s(1-c)}{p_b(1+c)} - 1 \Leftrightarrow \frac{p_t}{p_0} \geq \frac{p_s}{p_b} \tag{10}$$

Now let us assume the other strategy executes more than one trade. By the reasoning above, we can see that each trade has to has a ROI lower than that of the Buy and Hold strategy. Then the product of the ROIs of the other strategy also has to be lower than the ROI of the Buy and Hold.

As we can see, in order for a strategy to perform better than the Buy and Hold strategy the price needs to fall at some point. A strategy that sells when the price starts falling and buys if it rises again, will have a higher ROI.

3.4 Simple Moving Average Crossover

In this section we will see how the Buy and Sell signals are computed by the Simple Moving Average Crossover (SMAC) strategy. Then we will examine a model that generates closing prices based on several parameters. We will show that the SMAC strategy is optimal for this model.

3.4.1 The Strategy

A simple moving average (SMA) for a given time series is computed by averaging the last values up to some point. How many values are used depends on the sample size chosen for the SMA. For prices p_0 to p_t, the SMA f_t at point $t > 0$ with sample size $n > 0$ can be calculated as follows:

$$f_t = \frac{1}{n} \sum_{i=0}^{n-1} p_{t-i} \tag{11}$$

As we can see, if we have fewer than n prices (so when $t < n - 1$) the SMA can not be calculated. Thus, if we do want to inspect the strategy on fewer prices, we need to submit the remaining prices before p_0 to the strategy.

The SMAC strategy utilizes two SMAs with different sample sizes. The SMA with the smaller sample size is referred to by Technical Analysts as "fast". This is because they claim that it responds faster to price changes than a SMA with larger sample size. Conversely the SMA with the larger sample size is called "slow".

Let us define the fast SMA f_t as in Equation (11) above and the slow SMA s_t as:

$$s_t = \frac{1}{n+m} \sum_{i=0}^{n+m-1} p_{t-i} \tag{12}$$

where $m > 0$. This definition may look cumbersome, but in the next section when we construct the model, we will see how it provides itself useful.

The signal of the strategy is then given by the distance d_t between the SMAs:

$$d_t = f_t - s_t \tag{13}$$

A Buy signal is sent at time t, if the fast SMA crosses the slow SMA from below. In other words, this is the case if the distance is first negative and then zero or positive:

$d_{t-1} < 0$ and $d_t \geq 0$. Conversely a Sell signal is sent at time t, if the fast SMA crosses the slow SMA from above. We can formulate this in turns of the distance as well: $d_{t-1} > 0$ and $d_t \leq 0$.

Instead of computing the value of the distance d_t as in Equation (13), we can also compute it recursively, by computing the individual SMAs recursively:

$$
\begin{aligned}
f_t &= \frac{1}{n}\sum_{i=0}^{n-1} p_{t-i} \\
&= \frac{1}{n}(\sum_{i=0}^{n-1} p_{t-i-1}) + \frac{1}{n}(p_t - p_{t-n}) \\
&= f_{t-1} + \frac{1}{n}(p_t - p_{t-n}) && (14) \\
\Rightarrow s_t &= s_{t-1} + \frac{1}{n+m}(p_t - p_{t-n-m}) && (15) \\
\Rightarrow d_t &= f_{t-1} + \frac{1}{n}(p_t - p_{t-n}) - (s_{t-1} + \frac{1}{n+m}(p_t - p_{t-n-m})) \\
&= f_{t-1} - s_{t-1} + \frac{1}{n}(p_t - p_{t-n}) - \frac{1}{n+m}(p_t - p_{t-n-m}) \\
&= d_{t-1} + \frac{1}{n}(p_t - p_{t-n}) - \frac{1}{n+m}(p_t - p_{t-n-m}) && (16)
\end{aligned}
$$

This will slightly improve the run time of the algorithm, but most importantly it will make calculations in the next Section 3.4.2 easier.

Now we can define the algorithm that gives us a signal according to the SMA Crossover strategy.

Algorithm 2 Simple Moving Average Crossover Strategy

procedure SMACROSSOVER($[p_0, \ldots, p_t], n > 0, m > 0$)
    $f_{n+m-1} \leftarrow \frac{1}{n} \sum_{i=0}^{n-1} p_{n+m-1-i}$
    $s_{n+m-1} \leftarrow \frac{1}{n+m} \sum_{i=0}^{n+m-1} p_{n+m-1-i}$
    $d_{n+m-1} \leftarrow f_{n+m-1} - s_{n+m-1}$
    for $i \in [n+m, \ldots, t]$ **do**
        $d_i \leftarrow d_{i-1} + \frac{1}{n}(p_i - p_{i-n}) - \frac{1}{n+m}(p_i - p_{i-n-m})$
        if $d_{i-1} < 0$ **and** $d_i \geq 0$ **then**
            $signal_i \leftarrow 1$
        else if $d_{i-1} > 0$ **and** $d_i \leq 0$ **then**
            $signal_i \leftarrow -1$
        else
            $signal_i \leftarrow 0$
        end if
    end for
return $[0, \ldots, 0, signal_{n+m}, signal_{n+m+1}, \ldots, signal_t]$
end procedure

For each of the iteration of the for-loop, we perform a constant number of operations. Thus, the run time is in $O(t)$. We can see an example for this strategy in Figure 2.

3.4.2 The Model

In this section we will construct an asset-pricing model for which the SMA Crossover strategy is optimal. By assuming that the change in price is linear, we can show under which circumstances a Buy or Sell signal is sent. By adjusting the time until a signal is sent, we will get a model were the signals are sent at optimal points in time. First we examine the Buy signal. For this, we construct a price model which causes the strategy to send a Buy signal at time $b + 1$. We can then choose b in such a way,

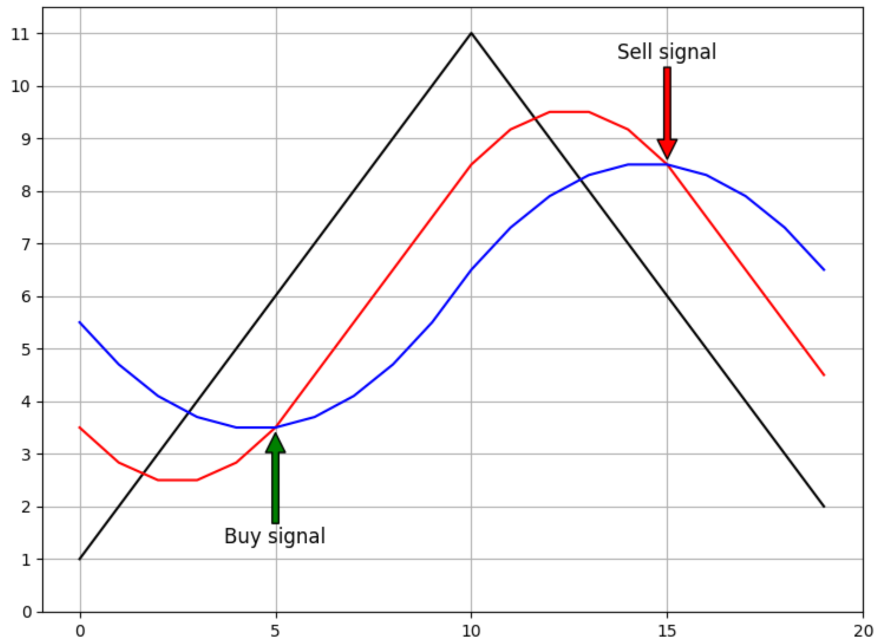

Figure 2: Example for the SMA Crossover strategy The black curve shows the price. The red curve is given by the value of a (fast) SMA with sample size 6. The blue curve is given by the value of a (slow) SMA with sample size 10. The arrows mark points in time, where a Buy or Sell signal is generated. At time 5, the fast (red) SMA crosses the slow (blue) SMA from below, generating a Buy signal. At time 15, the fast (red) SMA crosses the slow (blue) SMA from above, resulting in a Sell signal.

that a SMA crossover strategy sends the Buy signal at a better point in time than the Buy and Hold strategy.

Let us assume that the prices p_i for $i \in \{0, 1, \ldots, b\}$ and $b \geq 0$ are given by the following equation:

$$p_i = p_0 - i \cdot k \tag{17}$$

The price of an asset should not fall below zero. We can achieve this by setting the starting price $p_0 > b \cdot k$.
With these prices we can calculate the the value of the SMA f_b at point $b \geq n$:

$$\begin{aligned}
f_b &= \frac{1}{n}\sum_{i=0}^{n-1} p_{b-i} \\
&= \frac{1}{n}\sum_{i=0}^{n-1} p_0 - (b-i)k \\
&= p_0 - bk + \frac{k}{n}\sum_{i=1}^{n-1} i \\
&= p_0 - bk + \frac{n-1}{2}k
\end{aligned} \tag{18}$$

Equivalently, we get for the SMA s_b, where $b \geq n + m$:

$$s_b = p_0 - bk + \frac{n+m-1}{2}k \tag{19}$$

With the values for the fast and slow SMA, we can calculate the distance between them, for $b \geq n + m$:

$$\begin{aligned}
d_b &= p_0 - bk + \frac{n-1}{2}k - (p_0 - bk + \frac{n+m-1}{2}k) \\
&= (-m)\frac{k}{2}
\end{aligned} \tag{20}$$

As we have defined in the last Section 3.4.1, the difference between the sample sizes of the SMAs m has to be a positive integer. Thus, the distance d_b is negative. A Buy signal is sent, if the distance at the next point in time d_{b+1} is greater or equal to zero. We will use the recursive definition of the distance d from Equation (16) to show which value for price p_{b+1} is needed to create a Buy signal:

$$\begin{aligned}
&d_{b+1} \geq 0 \\
&\Leftrightarrow d_b + \frac{1}{n}(p_{b+1} - p_{b+1-n}) - \frac{1}{n+m}(p_{n+1} - p_{b+1-n-m}) \geq 0 \\
&\Leftrightarrow d_b + \frac{1}{n}(p_{b+1} - (p_0 - (b+1-n)k)) - \frac{1}{n+m}(p_{n+1} - (p_0 - (b+1-n-m)k)) \geq 0 \\
&\Leftrightarrow (p_{b+1} - p_0 + (b+1)k)(\frac{1}{n} - \frac{1}{n+m}) \geq -d_b \\
&\Leftrightarrow p_{b+1} \geq (-d_b\frac{n(n+m)}{m}) + p_0 - (b+1)k \\
&\Leftrightarrow p_{b+1} \geq p_0 + k(\frac{n(n+m)}{2} - (b+1)) \qquad (21)
\end{aligned}$$

In order to maximize the ROI, we want the lowest possible buying price p_{b+1} that still triggers a Buy signal.

$$p_{b+1} = p_0 + k(\frac{n(n+m)}{2} - (b+1)) \qquad (22)$$

From this equation, we can also calculate the change in price from p_b to p_{b+1}:

$$\begin{aligned}
p_{b+1} &= p_0 + k(\frac{n(n+m)}{2} - (b+1)) \\
\Leftrightarrow p_{b+1} &= p_0 - bk + k(\frac{n(n+m)}{2} - 1) \\
\Leftrightarrow p_{b+1} &= p_b + k(\frac{n(n+m)}{2} - 1) \\
\Leftrightarrow p_{b+1} - p_b &= k(\frac{n(n+m)}{2} - 1) \qquad (23)
\end{aligned}$$

We should note, that for sample size $n > 0$, and difference $m > 0$ the following holds:

$$\frac{n(n+m)}{2} \geq 1 \tag{24}$$

And thus the change of price $p_{b+1} - p_b$ has to be positive.
This strategy will outperform the Buy and Hold strategy, if the buying price p_{b+1} is lower than the starting price p_0:

$$\begin{aligned}
&p_0 - p_{b+1} > 0 \\
&\Leftrightarrow -k(\frac{n(n+m)}{2} - (b+1)) > 0 \\
&\Leftrightarrow b \geq \frac{n(n+m)}{2}
\end{aligned} \tag{25}$$

So if the price is falling in each time step by some value k, the smallest possible upwards change that still triggers a Buy signal is $k(\frac{n(n+m)}{2} - 1)$ (Equation (23)). If the point in time b is larger than $\frac{n(n+m)}{2}$ the Buy signal will be at a lower price than that of the Buy and Hold strategy.

Next we will examine the conditions under which this strategy sends a Sell signal. Let us assume again, that the prices p_i, for $i \in \{0, \ldots, s\}$ and $s \geq 0$ are given by a linear equation. Although now, the price is increasing:

$$p_i = p_0 + i \cdot k \tag{26}$$

With the same reasoning as in Equation (18) and Equation (20), we get:

$$f_s = p_0 + sk - \frac{n-1}{2}k \tag{27}$$

$$s_s = p_0 + sk - \frac{m+n-1}{2}k \tag{28}$$

$$d_s = m\frac{k}{2} \tag{29}$$

Because the slope k and the difference in sample size are positive ($m > 0$), the distance d_s has to be positive as well. In order to trigger a Sell signal at this point, the next distance d_{s+1} has to be lower or equal to zero. By using the same sequence of operations as in Equation (21), we get the required value for price p_{s+1}:

$$d_{s+1} \leq 0 \tag{30}$$

$$\Leftrightarrow p_{s+1} \leq p_0 - k(\frac{n(n+m)}{2} - (s+1)) \tag{31}$$

To maximize the ROI, we choose the highest price that triggers a Sell signal:

$$p_{s+1} = p_0 - k(\frac{n(n+m)}{2} - (s+1)) \tag{32}$$

We can calculate the change $p_{s+1} - p_s$ as in Equation (23):

$$p_{s+1} - p_s = -k(\frac{n(n+m)}{2} - 1) \leq 0 \tag{33}$$

The inequality holds, as we have seen in Equation (24).
In order to set the time s of the Sell signal, we need to put this part of the model together with the part describing the prices for a Buy signal.

We know from Equation (25), that the earliest buying time b with a buying price lower than price p_0 is $b = \frac{n(n+m)}{2}$. Thus, price p_b is given by:

$$p_b = p_{\frac{n(n+m)}{2}} = p_0 - \frac{n(n+m)}{2}k \tag{34}$$

We can calculate p_{b+1} by using Equation (23):

$$p_{b+1} = p_b + k(\frac{n(n+m)}{2} - 1) = p_0 - k \tag{35}$$

In order to achieve a higher selling than buying price, we need price p_{s+1} to satisfy $p_{s+1} > p_0 - k$. We can use Equation (32) to solve this condition after a small alteration. We asserted that the price needs to be rising before a Sell signal is sent, starting at some price p_0. We will now set this starting price to the buying price $p_{b+1} = p_0 - k$. Thus, we get one continuous asset-pricing model in which we have both a Buy and a Sell signal.
The last missing piece is the exact selling time $s + 1$, which we can determine as follows:

$$\begin{aligned}
&p_{s+1} > p_0 - k \\
&\Leftrightarrow (p_0 - k) - k(\frac{n(n+m)}{2} - (s+1)) > p_0 - k \\
&\Leftrightarrow sk - \frac{n(n+m)}{2}k > -k \\
&\Leftrightarrow s > \frac{n(n+m)}{2} - 1
\end{aligned} \tag{36}$$

Thus the earliest point s where a price change would trigger a Sell signal, resulting in a profitable trade, is at $s = \frac{n(n+m)}{2}$. Thus, by using Equation (33), the selling price p_{s+1} is given by:

$$
\begin{aligned}
p_{s+1} &= p_s - k(\frac{n(n+m)}{2} - 1) \\
&= p_0 - k + sk - k(\frac{n(n+m)}{2} - 1) \\
&= p_0 + \frac{n(n+m)}{2}k - \frac{n(n+m)}{2}k \\
&= p_0
\end{aligned} \tag{37}
$$

Thus, the strategy has an ROI of:

$$
ROI = \frac{p_{s+1}}{p_{b+1}} - 1 = \frac{p_0}{p_0 - k} - 1 > 0 \tag{38}
$$

Here, we can again see that the strategy outperforms the Buy and Hold on this model. The first and last price are equal to p_0, so the Buy and Hold has an ROI of 0. This is smaller than the SMAC ROI.

Now we have all prices of the model given. To conclude; we can calculate the prices given by an optimal asset-pricing model for the SMAC strategy with parameters $n > 0$ and $m > 0$ like so:

$$
0 \leq t \leq \lceil \frac{n(n+m)}{2} \rceil :
$$

$$
p_t = p_0 - tk \tag{39}
$$

$$
p_{\lceil \frac{n(n+m)}{2} \rceil + 1} = p_0 - k \tag{40}
$$

$$
p_{\lceil \frac{n(n+m)}{2} \rceil + 1 + t} = p_0 + (t-1)k \tag{41}
$$

$$
p_{n(n+m)+2} = p_0 \tag{42}
$$

$$
p_{n(n+m)+2+t} = p_t \tag{43}
$$

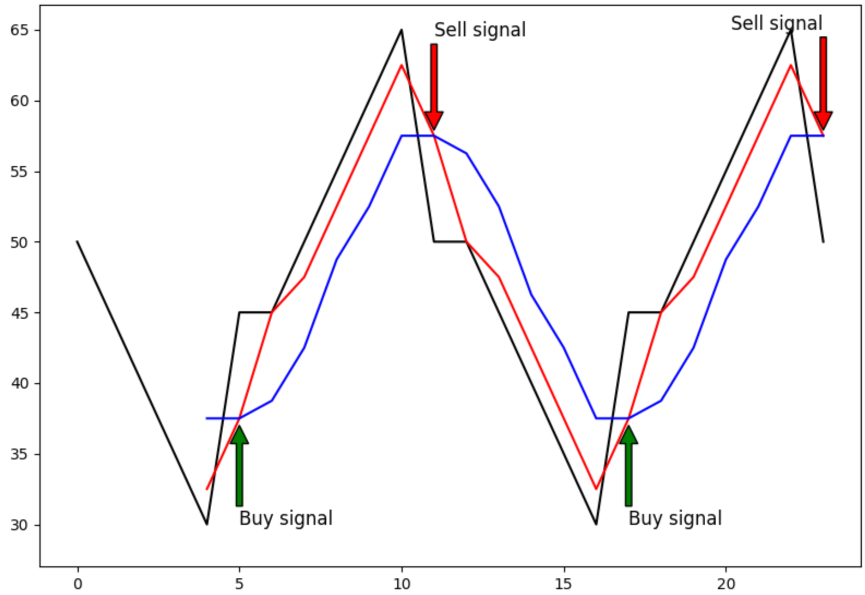

Figure 3: Example for the optimal SMA Crossover model The black curve shows the price. The red curve is given by the value of a (fast) SMA with sample size 2. The blue curve is given by the value of a (slow) SMA with sample size 4. The arrows mark points in time, where a Buy or Sell signal is generated.

The minimum price of this model is at time $\lceil \frac{n(n+m)}{2} \rceil$ and the maximum price is at time $n(n+m)+1$. The Buy signal is sent at the next point in time after the minimum, which is time $\lceil \frac{n(n+m)}{2} \rceil + 1$. Equivalently, the Sell signal is sent at the next point in time after the maximum, time $n(n+m)+2$. By our definition in Section 3.2, the SMAC strategy is optimal for this asset-pricing model. In Figure 3, we can an example of this model for a SMAC strategy with fast sample size $n = 2$ and slow sample size $n + m = 4$.

This model does not provide meaningful insights or a mathematical justification why this strategy should be useful. As Equation (43) shows, we need a ridiculously large number of prices to generate one Buy and one Sell signal. If we take for example some of the proposed parameters from Chapter 2, like $n = 50$ and $n + m = 100$. Then we would need $5,002$ prices. If we consider daily prices, this would mean roughly 14 years worth of price data for one trade. However, if this strategy would simply be made up and it has no better or worse performance than, for example, the Buy and Hold strategy, we are not able to do any better with another model. So in the next chapter (Chapter 4), we will test the hypothesis that the SMAC strategy does not perform significantly better than the Buy and Hold.

4 Empirical Analysis

In this chapter we will see the empirical Analysis of the strategies discussed in Chapter 3. After a description of the tools and sources used in the analysis, we will evaluate whether the SMA Crossover Section 3.4 (SMAC) strategy significantly outperforms the Buy and Hold Section 3.3 (BnH) strategy. This evaluation will be performed by using a one-tailed binomial test.

4.1 Applied Utilities and Data Sources

The evaluation was done using Python. The price data for Bitcoin originated from the Bitstamp exchange [14] and was then collected and published by Bitcoincharts.com [15]. The Ethereum price data was collected by Ethereumprice.org [16], who aggregate the prices across multiple exchanges. The stock prices were obtained via Alphavantage.co [17]. Unfortunately, Ethereumprice.org only provide daily price data and Alphavantage.co provides very small data sets below a daily granularity. Thus, for these assets, we will only evaluate the daily price history. Other reliable data sources for price history could not be found.

Bitstamp was used, because it is one of the longest standing cryptocurrency exchanges, with price data starting in the year 2015. However Bitstamp does not provide access to historic prices, only the current exchange rate. That is why we obtain the price data from Bitcoincharts. In this data set, some prices from the First of January, 2015 to the Ninth of January are missing. Thus the data set starts at the Ninth of January.

For data processing and calculating the signals of strategies the Pandas [18] library was used. The plots in this thesis were created using Numpy [19] and Matplotlib [20]. To compute the p-values, we use SciPy [21]

As we have discussed in Section 3.1, the strategies in this thesis only require the closing prices. Open, high, low and volume are not required. This suggests that it is sufficient to display only the closing prices as a curve, instead of the more common approach of visualizing OHLC-data with a candlestick chart.

4.2 The Binomial Test

We will perform a one-tailed binomial test in order to conclude whether the SMAC strategy outperforms the BnH strategy in a significant number of trials.

The samples will be randomly select from the complete data sets. We will assume, that a meaningful time period for an investment is at least three months (120 days) long. Also, if the last signal of the SMAC was a Buy signal and the next Sell signal is outside the sample period, we will assume (just like we did for the BnH) that all assets being held are sold at the end of the sample period.

Our null hypothesis H_0 is that the SMAC strategy has a higher ROI than the BnH with probability at most $\frac{1}{2}$. This would imply, that the SMAC strategy would be equivalent to randomly buying and selling, compared to the BnH strategy. The alternative hypothesis H_1, is then that the SMAC strategy outperforms the BnH strategy in a significant number of trials. In other words, it has a probability greater than $\frac{1}{2}$ of succeeding.

We can also express the hypotheses H_0 and H_1 in terms of the parameter $\theta \in \Theta$ from parameter space $\Theta = [0, 1]$ of the binomial distribution (with $n = 100$):

$$H_0 : \theta \in \left[0, \frac{1}{2}\right] \tag{44}$$

$$H_1 : \theta \in \left(\frac{1}{2}, 1\right] \tag{45}$$

To test this hypothesis, we perform $n = 100$ trials, which gives us the test statistic $x_1, \ldots, x_{100}$. If the SMAC outperforms the BnH strategy in trial i, we say that trial i has been successful and set $x_i = 1$. Else we say it was unsuccessful and set $x_i = 0$. If we set the significance level to $\alpha = 0.01$, we can reject the null hypothesis if:

$$\sum_{i=1}^{100} x_i > F^{-1}_{B(100,0.5)}(0.99) = 62 \tag{46}$$

Where $F^{-1}_{B(100,0.5)}$ is the inverse cumulative distribution function for a binomial distribution with parameters $n = 100$ and $p = 0.5$. Thus, we can reject the hypothesis if a SMAC strategy succeeds in at least 63 out of 100 trials.
Aside from the type I error probability α, we can also calculate the power of test $1-\beta$. Let us assume that we commit a type II error if we accept the null hypothesis even though the SMAC has probability at least 0.7 of outperforming the BnH ($\theta \geq 0.7$). Then committing this type of error has probability:

$$F_{B(100,0.7)}(62) = 0.053 \tag{47}$$

Thus we get the probability for a type II error $\beta = 0.053$ and power $1 - \beta = 0.947$. We will conduct this test for every reasonable setting of the SMAC strategy. This strategy has two inputs, aside from price data: the larger, slow SMA sample size (called $n + m$ in Section 3.4) and the smaller, fast SMA sample size (denoted by n in Section 3.4). In Section 2.4 we have seen that the highest reasonable setting for the large sample-size is a value of 200. Thus, we will test the SMAC for all possible parameter choices up to 300. The number of ROIs we have to compute is based on the large sample size, which we will call n' in this chapter (contrary to Chapter 3) in order to make the following equation clearer. For each choice of sample size n', there are $n' - 1$ possible choices for the small sample size. So, if we want to test all of these strategies up to some sample size n', the total number of possible SMAC strategies is given by:

$$\sum_{i=2}^{n'} i - 1 = \sum_{i=1}^{n'-1} i = \frac{n'(n'-1)}{2} \tag{48}$$

We can see that the number of possible strategies is quadratic in n'. For $n' = 300$, we get that we have to test $44,850$ strategies.

4.3 Strategy Evaluation on Bitcoin Prices

Our Bitcoin data set starts at the 9th of January, 2015 and ends at the 28th of February, 2020. Figure 4 shows this data set. This specific data set has daily prices, i.e. one price per day.

When we execute the binomial test on this data set, we get that there are 15 out of $44,850$ possible strategies that outperform the BnH strategy in a significant number of trials. Table 1 shows the p-values for these strategies. As we can see, none of the recommended parameters from Section 2.4 appear in this list. Thus, they do not cause a significantly better performance than the BnH. If we chose some parameters at random, we have a chance of 0.033% of picking a significantly better strategy than the BnH.

Aside from daily, other price granularities are examined by day traders, e.g. four hourly prices (one price every four hours). If we perform out test on this data set, we

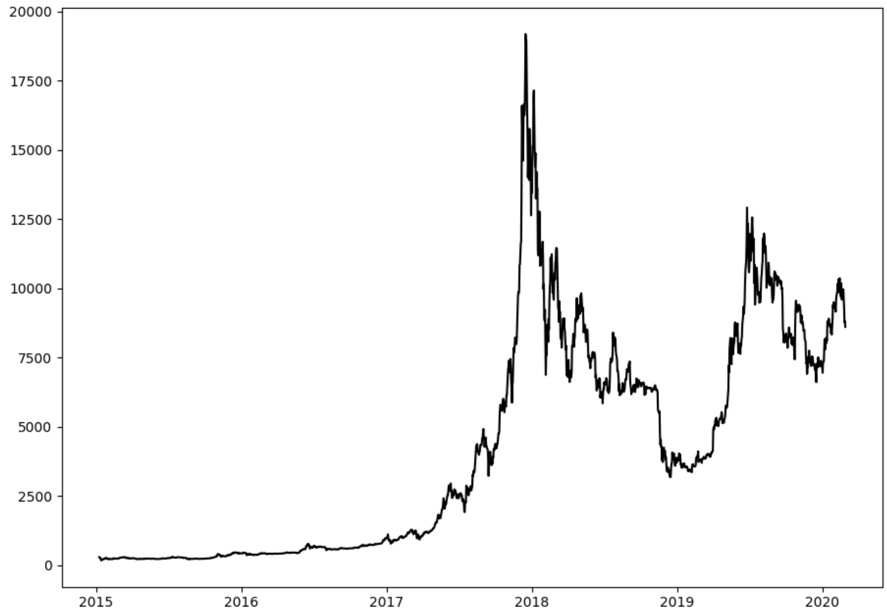

Figure 4: The Bitcoin price data set from which we pull random samples

find that there are no settings which significantly outperform the BnH. The lowest p-value we can find is 0.135 for parameters (144, 52).

4.4 Strategy Evaluation on Ethereum Prices

The Ethereum data set starts at the 7th of August, 2015 and ends at the 1st of March, 2020. As stated above, this is daily price data. For this data set we find, that $12,454$ out of $44,850$ outperform the BnH. Table 2 shows 20 SMAC settings which yield the lowest p-values. Note that no recommended parameters appear in this table.

For the recommended parameters, we find that there are several which cause the SMAC to outperform the BnH. Those parameters can be found in Table 3. Out of the 11 recommended settings, only 5 perform significantly better. We should note, that one of these better performing strategies is the one recommended by the "R+V

Slow SMA	Fast SMA	p-value
75	72	0.000016
75	71	0.000204
74	71	0.000437
76	74	0.000437
72	33	0.001759
73	70	0.001759
73	71	0.001759
74	70	0.001759
74	72	0.001759
72	32	0.003319
89	86	0.003319
76	75	0.006016
80	78	0.006016
83	79	0.006016
83	80	0.006016

Table 1: This table shows SMAC parameters and the corresponding p-values. The test was conducted as stated in Section 4.2. This list includes only settings with p-values smaller than 0.01.

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We can conclude: If we chose randomly one setting from the recommended ones, the chance of picking a profitable SMAC setting is about 45.45%. If we chose some parameter settings at random, the chance of picking a profitable one is around 27.77%.

4.5 Strategy Evaluation on Stock Prices

We will examine the price of one common stock and of one stock index: the Dow Jones. Although the Dow Jones Index (DJI) is not a tradeable stock it is useful for analysis as it provides an overview over the stock market. The common stock we will

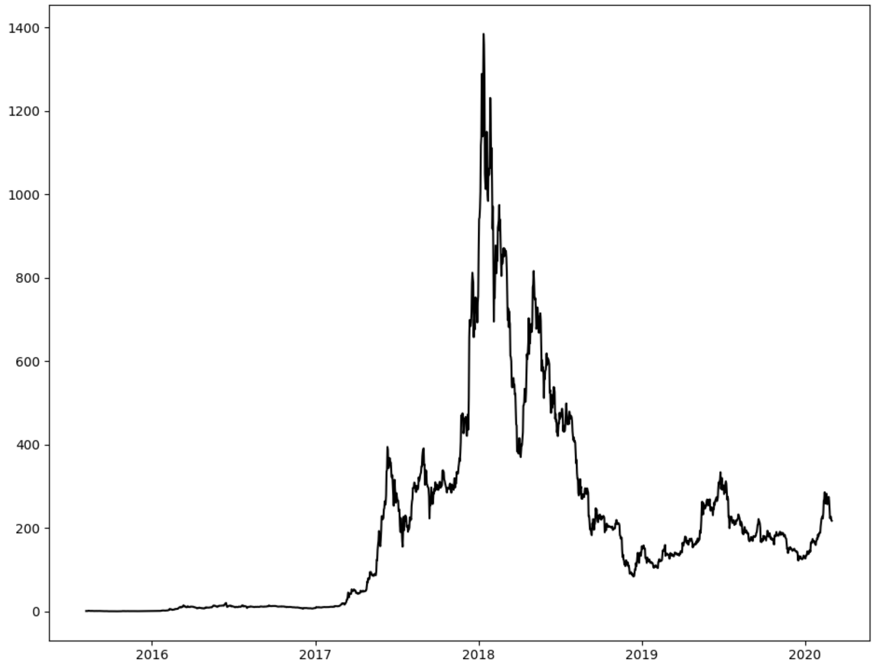

Figure 5: The Ethereum price data set from which we pull random samples

examine belongs to the company Microsoft (symbol: MSFT).
First we take a look at the DJI. The data starts at the 3rd of January, 2000 and ends at the 9th of March, 2020. The price data can be seen in Figure 6. We find that there 307 SMAC strategies which perform better than the BnH in a significant number of cases. The 20 best parameters can be found in Table 4. For all of these 307 strategies, the slow SMA parameter is greater or equal to 253 and the fast SMA parameter is greater or equal to 177. We find that none of the recommended parameters significantly outperform the BnH. The probability of randomly choosing a setting for the SMAC that is significantly better than the BnH is 0.68%.
The price data of the Microsoft stock starts at the 2nd of January, 2004 and ends at the 2nd of March, 2020. Figure 7 shows this data. For this data set, no parameters

Figure 6: The Dow Jones Index price data set from which we pull random samples

with p-values smaller than 0.01 could be found. The lowest p-value of 0.989 was achieved by a strategy with setting $(48, 147)$.

4.6 Strategy Evaluation on Foreign Currency Exchange Rates

We will examine the price data of the Euro (in USD). The data set starts at the 26th of February, 2002 and ends at the 9th of March, 2020. We can find $9,165$ settings which perform significantly better than the BnH. Table 5 shows 20 settings with the lowest p-values. Out of the recommended parameters, we find only two settings

Figure 7: The price data set based on the Microsoft stock from which we pull random samples

with a p-value smaller than 0.01: $(38, 200)$, with a p-value of 0.00601 and $(50, 100)$ with a p-value of 0.000204. Thus, if we had chosen one of the recommended settings randomly, the chance of picking a better performing one is about 18.18%. If we chose some setting randomly out of all that are possible, the chance of choosing better performing parameters is 20.43%. Thus it would be more reasonable to choose the parameter settings at random, instead of using the recommended ones.

Figure 8: The Euro price data set from which we pull random samples. The price of Euro is in US Dollars.

Slow SMA	Fast SMA	p-value
23	9	$1.302968 \cdot 10^{-12}$
25	8	$1.302968 \cdot 10^{-12}$
24	8	$6.548999 \cdot 10^{-12}$
20	13	$6.548999 \cdot 10^{-12}$
20	12	$3.073903 \cdot 10^{-11}$
23	10	$3.073903 \cdot 10^{-11}$
23	11	$3.073903 \cdot 10^{-11}$
22	10	$3.073903 \cdot 10^{-11}$
21	13	$3.073903 \cdot 10^{-11}$
29	8	$3.073903 \cdot 10^{-11}$
28	7	$3.073903 \cdot 10^{-11}$
31	5	$1.351381 \cdot 10^{-10}$
21	12	$1.351381 \cdot 10^{-10}$
21	14	$1.351381 \cdot 10^{-10}$
30	5	$1.351381 \cdot 10^{-10}$
31	4	$1.351381 \cdot 10^{-10}$
20	15	$1.351381 \cdot 10^{-10}$
21	10	$1.351381 \cdot 10^{-10}$
24	11	$1.351381 \cdot 10^{-10}$
24	10	$1.351381 \cdot 10^{-10}$

Table 2: This table shows 20 SMAC parameters with the lowest p-values for Ethereum. The test was conducted as stated in Section 4.2. In total, there were 12, 454 SMAC strategies that outperformed the BnH.

Slow SMA	Fast SMA	p-value
200	50	0.00175
100	20	0.00331
200	10	0.00331
200	38	0.00601
20	10	0.00895

Table 3: This table shows SMAC with recommended parameters from Section 2.4 sorted by p-value, computed on Ethereum prices.

Slow SMA	Fast SMA	p-value
270	187	0.000204
299	199	0.000437
273	186	0.000437
272	187	0.000437
269	189	0.000437
270	188	0.000437
271	187	0.000437
300	199	0.000437
269	188	0.000437
299	200	0.000437
298	200	0.000437
268	187	0.000437
268	188	0.000895
278	180	0.000895
299	195	0.000895
267	188	0.000895
267	187	0.000895
268	186	0.000895
268	189	0.000895
280	179	0.000895

Table 4: This table shows 20 SMAC parameters with the lowest p-values for the DJI. The test was conducted as stated in Section 4.2. In total, there were 307 SMAC strategies that outperformed the BnH.

Slow SMA	Fast SMA	p-value
142	52	$1.002980 \cdot 10^{-21}$
144	51	$1.002980 \cdot 10^{-21}$
143	51	$1.002980 \cdot 10^{-21}$
141	45	$1.363072 \cdot 10^{-20}$
141	44	$1.363072 \cdot 10^{-20}$
144	43	$1.363072 \cdot 10^{-20}$
142	45	$1.363072 \cdot 10^{-20}$
144	42	$1.363072 \cdot 10^{-20}$
140	46	$1.363072 \cdot 10^{-20}$
142	42	$1.363072 \cdot 10^{-20}$
146	47	$1.363072 \cdot 10^{-20}$
143	42	$1.363072 \cdot 10^{-20}$
143	43	$1.363072 \cdot 10^{-20}$
142	53	$1.363072 \cdot 10^{-20}$
143	45	$1.363072 \cdot 10^{-20}$
147	47	$1.363072 \cdot 10^{-20}$
142	43	$1.363072 \cdot 10^{-20}$
150	48	$1.363072 \cdot 10^{-20}$
143	50	$1.363072 \cdot 10^{-20}$
143	52	$1.363072 \cdot 10^{-20}$

Table 5: This table shows 20 SMAC parameters with the lowest p-values for the EUR/USD currency pair. The test was conducted as stated in Section 4.2. In total, there were 9165 SMAC strategies that outperformed the BnH.

5 Conclusion and Future Research

In this thesis, we have examined the sparse literature of sometimes questionable integrity, an optimal asset-pricing model for the Simple Moving Average Crossover (SMAC) strategy and results of an empirical test on asset prices.
Literature and the theoretical model were not of much help when confronted with the question why we should use the SMAC strategy. There is a definite lack of scientific literature and the unscientific literature we have seen does not provide any insights as well. The proponents of Technical Analysis (TA), and thus of the SMAC strategy, do not deliver valid arguments in favor of the strategy. This is especially embarrassing in case of the subsidiary of the Volksbank. An established financial institution is expected to do the necessary research on their products before selling them to customers who are for the most part lay people.

The theoretical analysis provided an asset-pricing model under which the SMAC strategy is in fact optimal and thus better performing than the Buy and Hold (BnH) strategy. However, this model seems very unrealistic in the context of asset prices. This means either that the model was poorly developed or that there simply is no model for which the SMA strategy is optimal and which closely describes the real world price data. Thus we hypothesised, that the SMAC strategy is not able to yield a higher return of investment (ROI) than the BnH in a significant number of trials.

In the empirical analysis we tested this hypothesis using a binomial test. We were able to find, that depending on the asset class, only up to 27.77% of parameter settings perform better than the BnH. This chance was calculated for Ethereum, most other assets have a much lower chance. Assume we are considering investing into one of these assets. If we had no other parameter recommendations, it would be more reasonable to use regular investing (i.e. the BnH strategy) instead of the SMAC. Because if we were to use the SMAC strategy to day trade, we would expect to choose a parameter setting for the strategy which yields a lower ROI than the BnH.

However, the self-proclaimed Technical Analysts and the "R+V Versicherung" recommend a set of parameters which define exactly 11 different SMAC settings. If we choose one of these 11 settings randomly, the highest chance of picking a better performing SMAC strategy is at most 45.45% in the case of Ethereum. For other assets, either none of settings caused the SMAC to outperform the BnH, or the chance of choosing a more profitable setting was even lower than when choosing parameters randomly.

Thus it would still be unreasonable to prefer the SMAC strategy over regular investing, even if we get a parameter recommendation by these "experts". This result is especially devastating for the "R+V Versicherung". Their recommendation of $(38, 200)$ was only found to perform better than the BnH for Ethereum and the Euro. However, they do not apply this strategy in the crypto or forex market, but instead in the stock market. For the Dow Jones Index (DJI), we did not find that their setting is better performing, even though they apply this strategy on a fund which structure is similar to that of the Dow.

The evidence we found does not suggest, that one should use SMAC strategy instead of making a regular investment. This is especially true for Bitcoin. However, there is still a lot of ground to cover for future research:

As we have seen, there are parameters for which the SMAC strategy performs significantly better than the BnH. Thus, the choice of parameters is very important to determine the usefulness of this strategy. The parameters suggested by TA do not appear to be reasonable, but there may be other means to determine a profitable setting for the SMAC strategy. We have seen that not only different asset classes require different parameters, but also different assets themselves. Developing an algorithm which is capable of determining the best SMAC setting for a given asset or price history would be sensible.

Another possible approach to future research would be to perform a more in-depth comparison between the different kinds of price data. One could for example investigate whether different settings are better suited than others for specific market sentiments (Bull or Bear markets). Another interesting comparison would be between different price granularities. Setting aside crypto currencies, one could also examine the performance between stocks from different sectors.

A more involved topic for future research would be to compare the ROI of the SMAC against that of the BnH. If the expected return for the SMAC is much higher than that of the BnH, it might be reasonable to use the SMAC, although we have a low chance of choosing the right parameters. The problem with this approach is, that we need to assume an asset-pricing model like the Black and Scholes in order to determine the expected ROI.

There are a lot of other strategies in TA. Applying the same approach as we did, future research could investigate whether there are other strategies with better performance than the SMAC. Other popular examples of TA strategies include the Relative Strength Index, the Bollinger Bands or the Stochastic Oscillator. But it should be noted, that they have the same lack of scientific justification as the SMAC strategy.

Finally, future research could also combine these approaches to an arbitrary degree. For example developing an algorithm which chooses parameters or even strategies based on the type of asset, data granularity, sentiment and so on.

We can conclude, that until evidence in favour of Technical Analysis is found, we can not recommend day trading over traditional investing. This includes automated trading systems (bots) which use TA-based strategies to make decisions. Our findings may also have consequences for research performed on more intricate strategies based on Machine Learning (ML). If only TA-based strategies are used as a baseline to compare ML strategy results or to train ML models, simple investing might still be more profitable than using a ML approach.

And perhaps the most important lesson of this research: before you trust financial advice always perform a one-tailed binomial test.

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